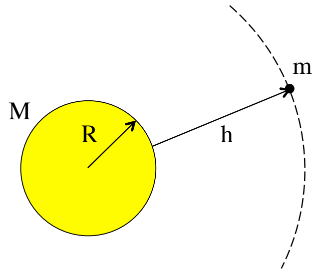
Problem 13.28

Consider the tiny satellite that is orbiting the earth of assumed mass M.



a.) How much potential energy does "m" have in its orbit?

The first thing to note is that the potential energy function for a mass NOT "close to the earth's surface" assumes a zero point where the gravitational force is zero, which is at infinity. Additionally, the origin from which "r" is measured is centered on the field-producing mass. That means the distance between the bodies isn't just the altitude, it's the altitude plus the earth's radius. With all that, we can write:

$$U(r) = -\frac{GmM}{(R+h)}$$

$$= -\frac{\left(6.67\text{x}10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.00\text{x}10^{2}\ \text{kg}\right)\left(5.98\text{x}10^{24}\ \text{kg}\right)}{\left(6.37\text{x}10^{6}\ \text{m}\right)+\left(2.00\text{x}10^{6}\ \text{m}\right)}$$

$$= -4.77\text{x}10^{9}\ \text{J}$$

NOTE: So what's the deal with the negative sign in front the of the potential energy function? Aside from the fact that that is what the derivation gave us, the conceptual answer is that orbiting systems are a *bound* system. That is, it takes *you* negative work to get the satellite in from infinity to its orbital distance. That means to get it back *out* to its zero level at infinity, *you* would have to do positive work. What the potential energy quantity is telling you is *how big an energy hole you are in*.

b.) What is the magnitude of the gravitational force exerted on the satellite by the earth?

$$\left|\vec{F}\right| = \frac{GmM}{(R+h)^2}$$

$$= \frac{\left(6.67\text{x}10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.00\text{x}10^{2}\ \text{kg}\right)\left(5.98\text{x}10^{24}\ \text{kg}\right)}{\left[\left(6.37\text{x}10^{6}\ \text{m}\right)+\left(2.00\text{x}10^{6}\ \text{m}\right)\right]^2}$$

$$= 569\ \text{N}$$

c.) What force does the satellite exert on the earth?

By N.T.L., the force the earth exerts on the satellite will be equal and opposite the force the satellite exerts on the earth.